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Carderock Division
Naval Surface Warfare Center
Bethesda, MD 20084-5000

CARDEROCKDIV-SME-92/25 May 1992
Ship Materials Engineering Department
Research and Development Report

3-D Nonlinear Constitutive Modeling Approach for Composite Materials

by
Karin L. Gipple

CARDEROCKDIV-SME-92/25 3-D Nonlinear Constitutive Modeling Approach for Composite Materials

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Abstract

This program outlines a three step process to develop nonlinear composite material models, and incorporate those models in a general, large scale structural finite element analysis code. The steps include detailed finite element analyses at the micromechanical (fiber-matrix) level to study material response and nonlinear mechanisms, development of a simplified material model that is representative of the behavior observed in the micromechanical analyses, and the inclusion of the simplified model in the large scale finite element analysis. Only the first two steps are addressed in this paper. Micromechanical finite element analyses of AS4/3501-6 and S2/3501-6 determined stress-strain responses for in-plane and transverse shear, transverse tension and transverse compression. Variation of the predicted behavior with failure criteria and comparisons to experimental data were also shown. The performance of a simplified material model based on a unit cell analogy was compared to the behavior observed in the micromechanical analyses.

Administrative Information

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INTRODUCTION

Composite materials can exhibit a significant nonlinear stress-strain response due to a variety of mechanisms including material nonlinearities, damage, and interfacial debonding [1]. These nonlinearities must be considered for accurate prediction of strength or stability based failure of thick composite structures. Through thickness stresses can not be ignored in these structures, and significant material nonlinearities are expected for these matrix dominated stress states.

It is suspected that linear material assumptions have

several effects on the results of structural analyses. Inaccurate predictions of stresses, strains, deflections or buckling may occur if material softening is neglected. These effects are often counter intuitive with composite materials. On the one hand, a nonlinear structural analysis may result in more critical stresses, strains or deflections than a linear analysis. On the other hand, stress singularities predicted with linear material properties may be relieved when the nonlinear material behavior is included in the structural analysis [2,3].

This report documents the development of a three-dimensional (3-D) nonlinear constitutive modeling approach for composite materials. The nonlinear material model is based on a combination of experimental results and micromechanical finite element analysis results. Incorporation of the material model into a structural analysis is the result of a three step process:

1) development of the detailed 3-D nonlinear response using micromechanical finite element analyses
2) formulation of a simplified material model whose behavior matches the detailed analysis results
3) transition of the model to a general purpose structural analysis finite element program.

BACKGROUND

Early efforts in composite nonlinear material modeling focused on 2-D models governed by continuum mechanics

formulat·ons. Most of these models were based on nonlinear elasticity theory [4,5,6]. Other approaches included classical incremental plasticity theory [7] and endochronic plasticity [8]. Griffin et al. [3] extended the effort to three dimensions using Hill's anisotropic formulation. Micromechanical techniques were also developed to address issues ignored by continuum mechanics [9,10]. These methods provided detailed information on matrix and fiber stress distributions for any combination of loading. In addition, they could be used to evaluate the effect of constituent behavior on the overall composite response.

Although recent works have built on earlier approaches in classical plasticity [1] and endochronic plasticity [11], micromechanics continues to be a strong area of interest due to the insights it offers into composite material behavior. Sun and Chen [12], Aboudi [13] and Adams [14] have developed unit cell models of varying degrees of complexity to represent a repeating unit within a unidirectional lamina. These models are then used to predict the nonlinear behavior of a laminate. Sun and Chen acknowledged that typical micromechanical models are too complex to use in the analysis of composite structures. They used results (elastic-plastic stress-strain curves for various loading combinations) from their detailed micromechanical model to develop a simpler, one parameter plasticity approach for incorporation in a structural analysis. The study described in this paper uses a similar approach where a simplified unit cell model

developed by Pecknold [15] is forced to match the behavior observed in detailed micromechanical finite element analyses.

MATERIAL MODEL DEVELOPMENT STRATEGY

Several requirements were established for the material model in this program. First, the model must be suitable for use in typical composite structural analysis tools. This feature provides a means to evaluate material constitutive effects on structural performance. However, incorporation of a material model into a large scale structural finite element program limits the complexity of the material model due to the large number of times the material model is invoked in these analyses [12,15]. A complex material model would result in prohibitive computer run times.

Secondly, the nonlinear behavior must be characterized in three dimensions for any given loading condition by the model. Extension beyond the 2-D characteristics of classical lamination theory (CLT) is necessary so that accurate out-of-plane material behavior can be included in structural analyses. Characterization of the overall nonlinear behavior covers plasticity and damage effects, both of which could have significant impact on structural analysis results [1,2,3].

Evaluation of the stress-strain response for any given loading condition is extremely difficult to accomplish experimentally due to the inherent problems in applying

combined loads on material test specimens. And since the material behavior is nonlinear, the individual uniaxial responses cannot be superimposed. Material response under multiaxial load conditions is particularly useful for structural analyses as the critical areas are often subject to complex stress states. A model that can provide insight into the nonlinear material response for any loading combination can be used in lieu of testing.

The preceding requirement suggests the use of a micromechanical material model where the constituent (fiber and matrix) properties can be related to the composite properties. Any loading combination can be applied to the micromechanical model which is then used to generate the desired composite response [12,13,15].

These requirements led to the development of a 3-D nonlinear material modeling scheme that is conceptually similar to that used by Sun and Chen [12] for metal matrix composites. It consists of three steps: development of the detailed 3-D nonlinear response, formulation of a simplified material model, and transition of the model to a general purpose, structural analysis finite element program, such as ABAQUS [16], through a user-written material (UMAT) subroutine.

Detailed Micromechanical Analyses

Micromechanical methods and selective experimentation are used to develop an overall understanding of the 3-D

material inelastic response. In the micromechanical approach, composite behavior is analyzed using some type of idealized model of the individual fibers and surrounding matrix material [14]. The idealized model may be as simple as a Rule of Mixtures (ROM) formulation [17] or as complex as a finite element analysis of a representative volume of a unidirectional composite. The latter methodology was chosen for this program as it most closely models the actual composite.

WYO2D, a 2-D finite element analysis program developed by the Composites Materials Research Group in the Department of Mechanical Engineering at the University of Wyoming, was selected for the micromechanical finite element anaylses. This program is tailored for micromechanical analyses of continuous fiber reinforced composite materials and incorporates many desirable modeling and material features. Since the source code is provided, the program can be easily modified by the user.

WYO2D represents a unidirectional composite as a collection of unit cells consisting of a fiber embedded in a matrix cube. Using a generalized plane strain approach, this 3-D unit cell is reduced to a 2-D case. Due to symmetry conditions, only one quadrant of the 2-D unit cell representation needs to be modeled (see Figure 1). A sample mesh, including fiber, matrix and interface elements, is shown in Figure 2 [18]. Constant strain triangles are used

for computational speed and the finite element grid is automatically scaled to reflect changes in fiber volume fraction.

Although a rectangular packing array is assumed for the fibers, any fiber packing geometry can be accomodated. Voids, disbonds, defects and degraded interfaces can also be modeled. Using a technique detailed in Reference [14], displacement boundary conditions are applied to allow the simultaneous application of any combination of loads. Temperature and/or moisture content changes can be induced to represent such phenomena as cooldown induced residual stresses, moisture induced swelling stresses, and moisture induced constitutive changes in the matrix material.

Constituent modeling features allow the use of anisotropic fiber and elastoplastic matrix properties. The plastic response of the matrix is governed by the Prandtl-Reuss flow rule which assumes that plastic strain is proportional to the deviatoric stress [14,19]. The constitutive equations developed with this flow rule require expressions for the octahedral shear stress-strain behavior. These expresssions are obtained from a Richard-Blacklock curve fit of the matrix shear stress-strain data as shown in Equation 1.

$$\tau = (G\epsilon)/[1 + (G\epsilon/\tau_o)^n]^{1/n} \qquad (1)$$

τ = shear stress
ϵ = shear strain
G = initial tangent shear modulus
τ_o, n = curve fitting parameters

Various failure criteria including the octahedral shear stress, maximum normal and maximum shear [20] can be chosen by the user in WYO2D to determine element failures. If the element stress(es) exceeds the specified criteria, the element stiffness is reduced to near zero and the strain energy of the failed element is redistributed through nodal forces.

Small modifications to the program were made by the authors. An option to ignore any failure criteria in the analysis was included to look solely at matrix plasticity effects on the overall nonlinear stress-strain response. In addition, the interface material's tensile strength was set equivalent to 93% of the matrix dry strength, which is halfway between the matrix's room temperature dry and wet strengths. This interfacial strength was chosen based on scanning electron microscopy (SEM) observations of interfacial failures in wet and dry 3501-6 composites. The fiber and matrix material properties used for analysis purposes are listed in Tables 1 and 2.

Simplified Material Model (Spring Model)

Within a larger scale finite element analysis, material model calculations must be performed at each integration point for all the elements. This occurs for every iteration (to convergence) within a load increment over the total number of load increments. A material model, such as WYO2D, that is, in itself, a detailed finite element analysis would

make the process computationally infeasible. Therefore, a technique was required to summarize the detailed micromechanical results in a simplified form so that reasonable computation times were achieved.

This was accomplished using a material model, outlined by Pecknold [15], whose behavior was forced to resemble that of the micromechanical analyses performed with WYO2D. In Pecknold's approach, a simplified unit cell and stress averaging techniques are used to develop the response of a unidirectional lamina. The spring analogies to the unit cell configuration in Figure 3 are based on the variations in load path for in-plane longitudinal loads and shear/out-of-plane loads. Average stresses and strains within the unit cell are related by the tangent stiffness for the unit cell. The tangent stiffness is a combination of the tangent stiffness parameters of the fiber and resin components of the unit cell.

As in WYO2D, the fiber is considered transversely isotropic. In contrast to the elastoplastic matrix material model in WYO2D, the matrix shear behavior is nonlinear elastic and expressed in a Ramberg-Osgood [21] form.

$$\epsilon = \tau/G + \beta(\tau_o/G)(\tau/\tau_o)^N \qquad (2)$$

ϵ = shear strain
τ = shear stress
G = initial tangent shear modulus
β, τ_o, N = curve fitting parameters

The material constants used in the spring model are listed in Tables 1 for the fiber (same as WYO2D) and Table 3 for the 3501-6 epoxy matrix.

Unidirectional lamina within the spring model are assembled into sublaminates, the smallest repeating unit of the overall laminate. The conditions governing the assembly are based upon classical laminate theory with one extension. The out-of-plane stresses at each lamina interface must be continuous across the interface to satisfy equilibrium [22,23]. Development of the sublaminate tangent stiffness is dependent upon smearing techniques where the inhomogeneous laminate is replaced by a homogeneous anisotropic material.

Finite Element Computations

Within a large finite element program, a material subroutine is typically supplied with vectors containing total stresses and strains at the beginning of the load increment, and strain increments. Using this information, incremental stress updates must be determined within the subroutine. The algorithm chosen for these updates should be computationally efficient due to the large number of times it is executed in the analysis.

This study used an algorithm written by Hajali [25]. The algorithm increments strains directly by taking the strain at the previous increment and adding on the strain increment. Both the strain at the previous increment and the strain increment are provided as input to the material subroutine

from the main finite element program. Micro level stress and strain increments are approximated using global stress and strain increments and tangent material properties. The micro level stress and strain increments are used to calculate new strain totals which must be adjusted in an iterative process (modified Newton-Raphson method) to satisfy equilibrium, compatability and nonlinear constituent equations. Future refinements of the stress update procedure are expected to improve computational speed and convergence time.

RESULTS AND DISCUSSION

Detailed Micromechanics Model Results: Uniaxial Loading

Four uniaxial load cases were studied using WYO2D for unidirectional AS4/3501-6 and S2/3501-6 laminates. These included in-plane shear, transverse shear (2-3 plane shear), transverse tension and transverse compression as shown in Figure 4. For each load case, the stress-strain behavior to failure was predicted using different failure criteria (octahedral, maximum normal stress, and maximum shear stress). These behaviors were compared to the response predicted with no failure criteria, i.e. assuming material plasticity effects only. Figures 5 through 12 show all the generated stress-strain responses.

When available, test data was superimposed upon the predicted behavior. Figures 13 through 16 include in-plane and transverse shear data from the Iosipescu V-notched beam

specimen in the modified Wyoming test fixture [26] and/or $\pm 45^{o}$ tension [27] test for unidirectional AS4/3501-6 and S2/3501-6 laminates. Each of these methods is commonly used to determine the shear modulus in the linear range. The Iosipescu specimen is also used for shear strength determination. The measured nonlinear response is quite different for each specimen.

No attempt was made to force the predicted behavior to match either set of data as has been commonly done in other micromechanical approaches to account for uncertainties in the parameters describing the fiber and matrix. The predicted response is based on a unit cell subjected to a pure and uniform shear load. In reality, the Iosipescu and $\pm 45^{o}$ tension specimens have a more complex stress state due to geometry, loading effects, and damage [28]. The data and predictions may not match depending on the severity of these effects. If the predicted response is considered representative of the true material response, then structural analyses of the shear specimens using the nonlinear material model should match the data in the region where the strain gages were mounted. Of course, this is contingent on accurate loadings and boundary conditions in the structural analysis. The correlation of these analyses with specimen data will be documented in two ensuing reports.

Transverse tension data was obtained at DTRC using an IITRI specimen [29]. Data is available for S2/3501-6 only.

As seen in Figure 17, measured and predicted initial tangent moduli correlate well, but the data shows more softening than predicted.

Figures 18 and 19 show transverse compression data from Reference 30 for AS4/3501-6 and S2/3501-6. The predicted octahedral response correlates well with the AS4/3501-6 data until just prior to failure. The transverse compression data for S2/3501-6 is highly nonlinear, more so than any of the predicted responses.

In the preceding comparisons of predicted responses and experimental data, it appears as if the choice of failure criteria is dependent on the particular load case. However, these comparisons must be interpreted carefully. Direct comparisons are valid only to the extent that the stress state in the specimen gage section is uniaxial and uniform. Stress analyses of the specimens incorporating both material nonlinearities and damage induced nonlinearities would yield more accurate comparisons.

Detailed Micromechanics Model Results: Multiaxial Loading

Any combination of loads can be applied to a finite element unit cell model to assess the effect of multiaxial loading on strength and/or stress-strain behavior. The difficulty comes in obtaining multiaxial test data for verification. One comparison of data and predicted response was made using experimental data obtained by Davis [31] on boron/epoxy tubes subjected to combined shear (torque) and

axial compression. Shear stress-strain curves were with the tubes under three levels of axial compression (0, 1048, and 1572 MPa). Theoretical simulations of this load state were done by applying in-plane shear, transverse shear and axial compression loads to the unit cell. The transverse shear loads represented a component of the axial compression load on the tube that is generated due to imperfections such as fiber waviness. The magnitudes of the transverse shear loads were calculated numerically by Davis. Figure 20 shows the predicted and experimental stress-strain curves.

Simplified Material Model Results: Uniaxial Loading

Initially, the Ramberg-Osgood parameters describing the matrix in the spring model were chosen to exactly match the Richard-Blacklock model of the matrix shear behavior in WYO2D. The predicted matrix shear response for each model is shown in Figure 21. The in-plane shear response for a unidirectional lamina using the spring model was then compared to that generated with the detailed finite element analysis (WYO2D). The spring model approach was significantly stiffer as shown in Figure 22. The parameters describing the matrix within this unit cell were then adjusted so that the predicted in-plane shear response matched the predicted in-plane shear response of the detailed FEA performed with WYO2D for both AS4/3501-6 and S2glass/3501-6. Figure 23 compares the shear behavior of the matrix constructed with these parameters to the matrix in

WYO2D.

The best correlation between the spring model and FEA results for in-plane shear was obtained with β=1, τ_0=106 MPa, N=5 and G=1.886 GPa as shown in Figures 24 and 25 for graphite and glass/epoxy. Using the same parameters, comparisons of the transverse shear, transverse tension and transverse compression responses (unidirectional lamina) using the spring model and WYO2D are shown in Figures 26 through 31.

The AS4/3501-6 behavior predicted by the spring model correlates well with the micromechanical results for each load case without a change in spring model parameters from load case to load case. This is not observed for the S2/3501-6. The parameters that are used to correlate the in-plane shear behavior do not result in matched behavior for the remaining load cases. No attempt was made to address this discrepancy for the S2/3501-6.

Finite Element Computations

Structural analyses performed with ABAQUS using the described nonlinear material models will be documented in later reports. The cases studied include in-plane shear specimens, transverse shear specimens and composite cylinders.

CONCLUSIONS

An approach has been implemented to incorporate 3-D nonlinear material behavior into structural analyses. The three steps of this approach include development of the detailed 3-D nonlinear response using micromechanical finite element analyses, formulation of a simplified material model, and transition of the model to general purpose structural analysis finite element programs. These steps provide a range of methods to evaluate nonlinear behavior from the study of nonlinear mechanisms at the fiber-matrix level to the assessment of nonlinear material effects on structural performance.

Detailed finite element micromechanical analyses form the basis for the constitutive model in this program. They offer a unique capability to evaluate response to combined loadings and to incorporate the use of failure theories to evaluate the effect of damage on composite material nonlinearity. Comparison of analysis results with available uniaxial data showed good correlation for AS4/3501-6 laminates loaded in transverse compression, but not for S2glass/3501-6 laminates. However, transverse tension data for S2glass/3501-6 compared favorably with the predicted behavior. Correlation of analysis with in-plane and transverse shear data will be done in ensuing reports where specimen structural analyses are performed with the nonlinear material models developed here. The results of the micromechanical analyses match limited experimental data for combined compression and shear loading.

Rigorous finite element analyses at the fiber-matrix level are too complex to interface directly to structural finite element analysis codes. A simplified material approach called the spring model was evaluated for this purpose. The performance of the spring model correlated well with AS4/3501-6 micromechanical analysis results, but less so with the S2glass/3501-6 results.

A subroutine incorporating the spring model is available for ABAQUS and/or similar finite element analysis programs. Optimization of the stress updating procedure within the subroutine may be necessary to increase computational efficiency.

FUTURE WORK

It was desirable to incorporate this constitutive model in a larger scale finite element program as quickly as possible to determine the material nonlinearities critical to structural performance. Based on this decision, WYO2D and Pecknold's spring model were chosen as they were readily available, flexible tools. Other options included using comparable micro-macro material models or writing our own. The use of currently available tools allowed us to focus on critical unknowns. There are several areas where additional work is expected. The first is in micromechanics. It is likely that the failure criteria in WYO2D will be replaced by criteria more suitable to the particular matrix system. For example, a modified Mohr's failure criteria [20] for brittle

materials with different tensile and compressive strengths may be ideal for modeling the 3501-6 epoxy system. This failure criteria is more complex, in itself, requiring a numerical solution. In addition, the plasticity model may be changed to reflect new materials.

Other ongoing programs are generating experimental data on strength and failure mode variations in wet and dry composite materials. The insight provided in these programs on matrix and interfacial influences will be incorporated into the detailed micromechanical analyses.

Another area of concern regards the simplified unit cell model. It is possible that this model may not track some critical behavior, particularly the response to multiaxial loadings. Methods of incorporating failure or damage into this model must also be considered.

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PROPERTY	FIBER	
	AS4	S2GLASS
E_1 (GPa)	224	88
E_2 (GPa)	14	88
E_3 (GPa)	14	88
ν_{12}	.2	.22
ν_{13}	.2	.22
ν_{23}	.25	.22
G_{12} (GPa)	14	35
G_{13}(GPa)	14	35
G_{23}(GPa)	5.6	35

Subscripts indicate direction

1 Along fiber

2,3 Transverse to fiber

Table 1. AS4 graphite and S2 glass fiber properties used in analysis

Property	Epoxy Resin 3501-6 Room Temperature, 0% Moisture
E (GPa)	4.3
G (GPa)	1.9
ν	.34
n	1.604
τ_0 (MPa)	160

Richard-Blacklock Shear Model

$$\tau = \frac{G\gamma}{[1 + (G\gamma/\tau_0)^n]^{1/n}}$$

τ Shear stress

γ Shear strain

G Initial tangent shear modulus

n, τ_0 Curvature parameters

Table 2. 3501-6 epoxy properties used in finite element (WYO2D) micromechanical modeling

Property	Epoxy Resin 3501-6 Room Temperature, 0% Moisture
K (GPa)	2.0
G (GPa)	1.9
β	1.0
N	5.0
τ_0 (MPa)	106

Bulk Modulus

$$K = E / (9 - 3E / G)$$

Ramberg Osgood Shear Model

$$\gamma = \tau/G + \beta(\tau_0/G)(\tau/\tau_0)^N$$

τ Shear stress

γ Shear strain

G Initial tangent shear modulus

β, N, τ_0 Curvature parameters

Table 3. 3501-6 epoxy properties used in spring model

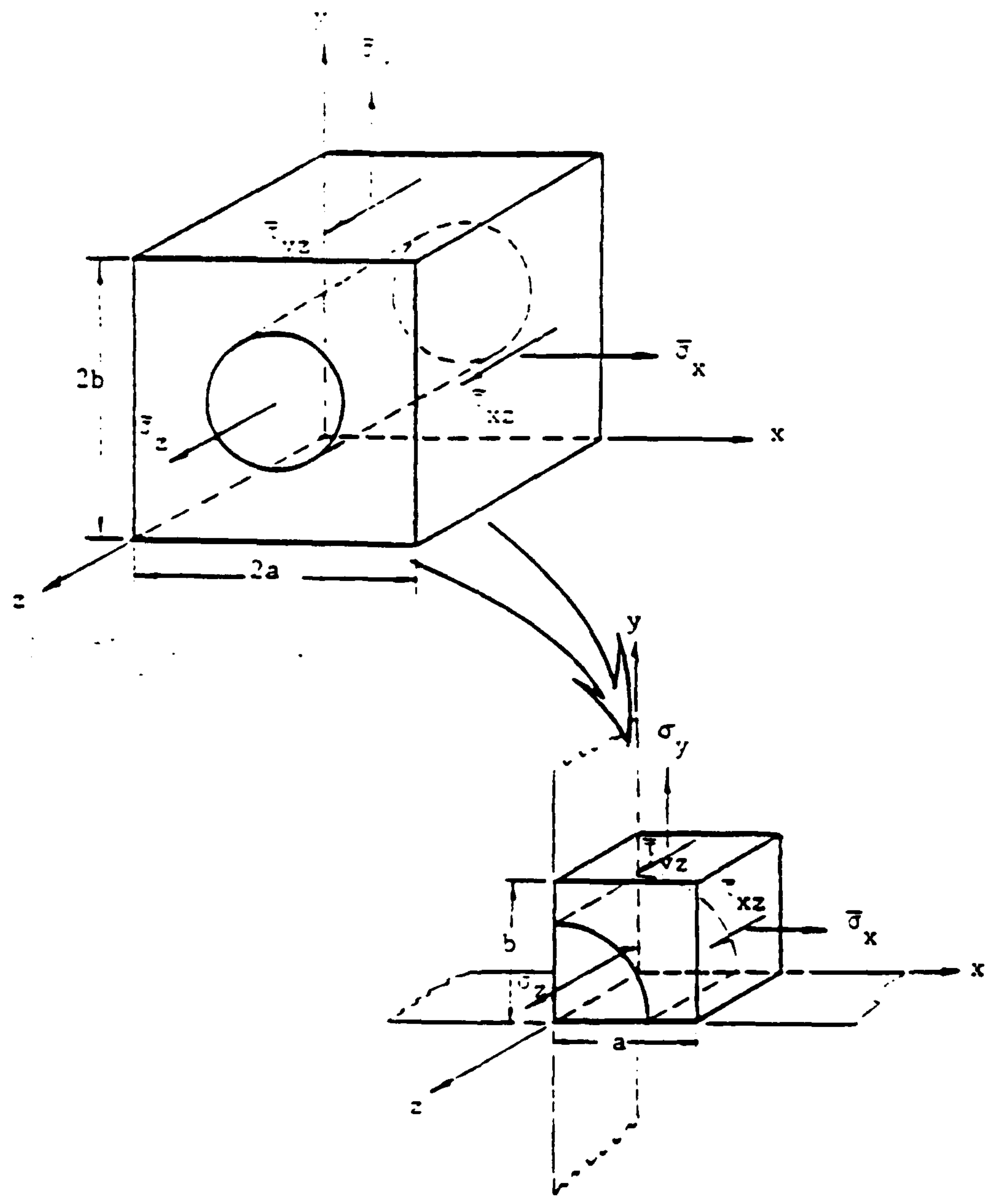


Figure 1. WYO2D unit cell representation of fiber and matrix (after reference 14)

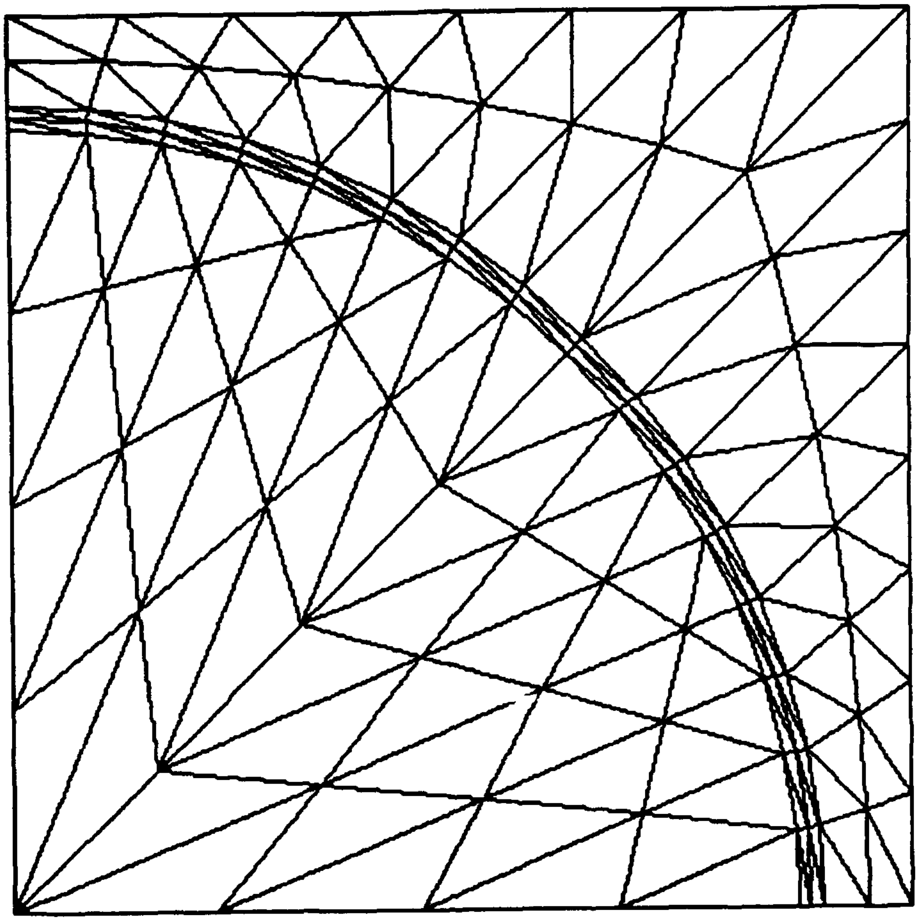


Figure 2. Finite element mesh of WYO2D unit cell quadrant (after reference 18)

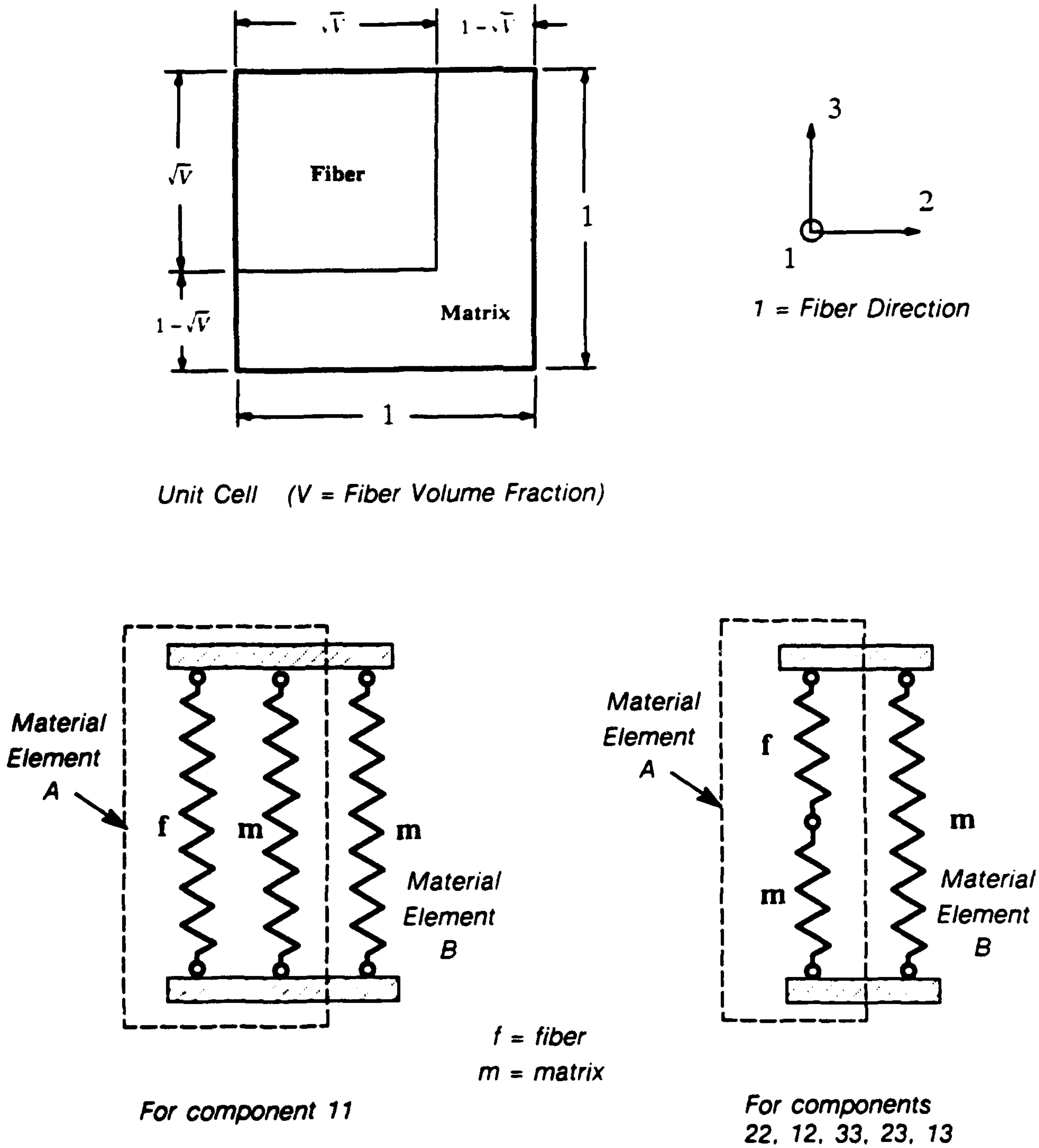
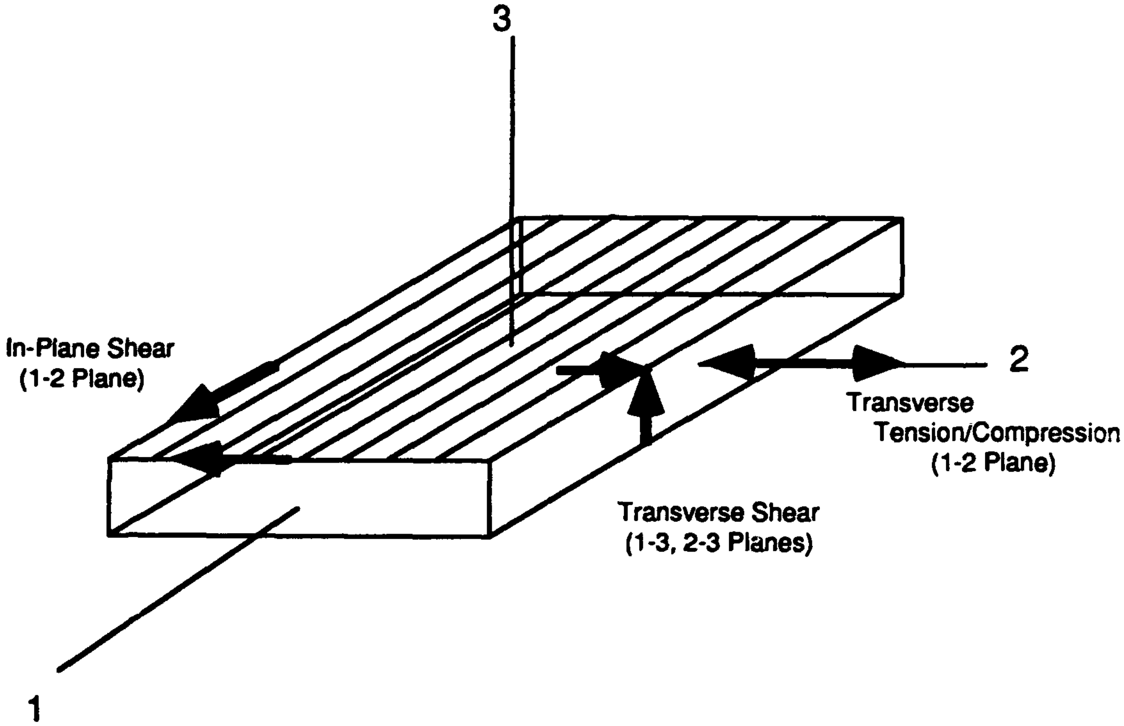


Figure 3. Spring model unit cell analogy showing load path variations between fiber and matrix dominant components



Directions

1	Along fiber
2,3	Transverse to fibers

Figure 4. Micromechanical model loading directions

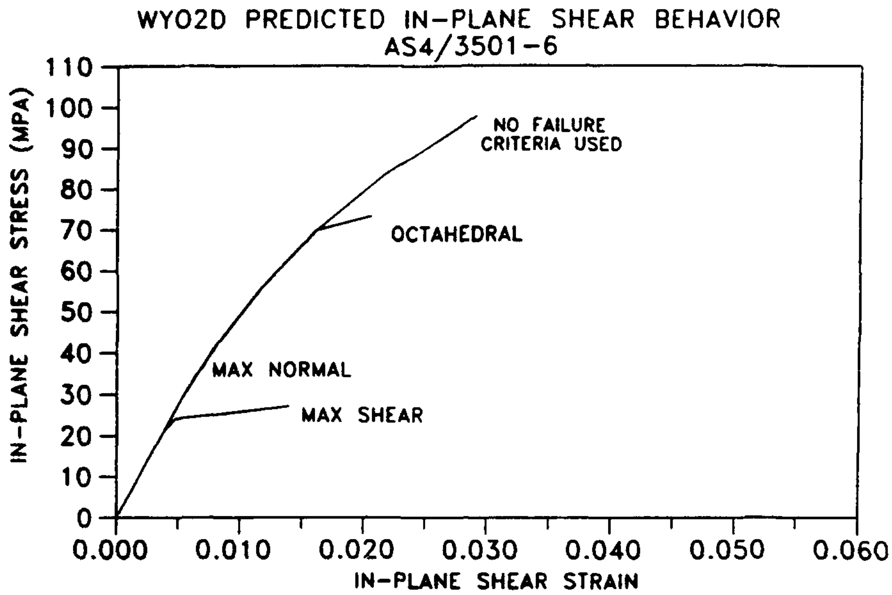


Figure 5. **WYO2D predicted in-plane shear behavior for AS4/3501-6 using no failure criteria (material plasticity effects only), and the octahedral, maximum normal stress, and maximum shear stress failure criteria**

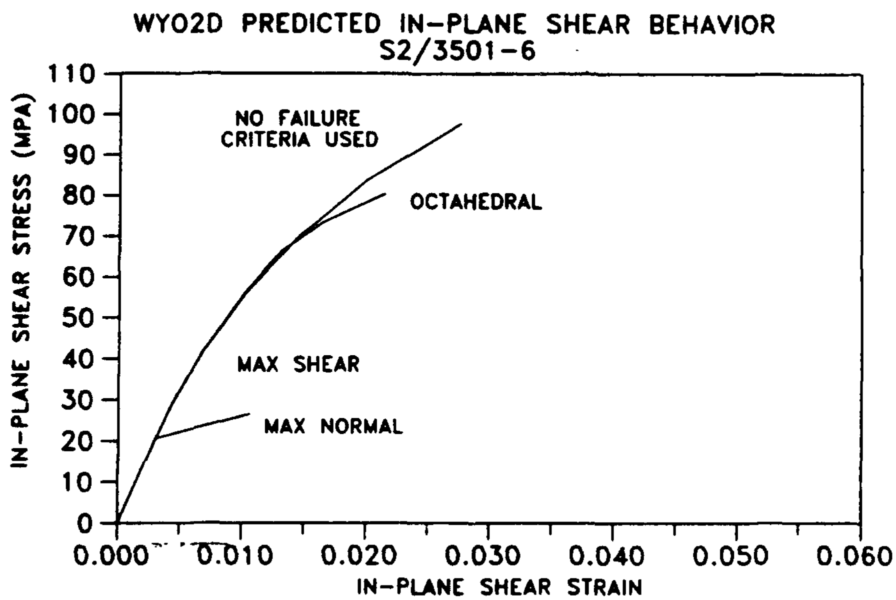


Figure 6. **WYO2D predicted in-plane shear behavior for S2glass/3501-6 using no failure criteria (material plasticity effects only), and the octahedral, maximum normal stress, and maximum shear stress failure criteria**

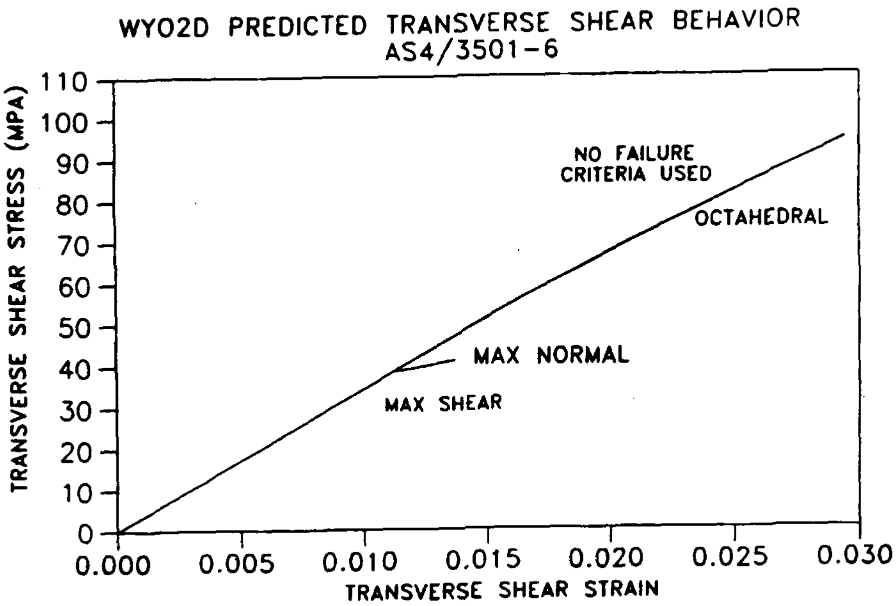


Figure 7. WYO2D predicted transverse shear behavior for AS4/3501-6 using no failure criteria (material plasticity effects only), and the octahedral, maximum normal stress, and maximum shear stress failure criteria

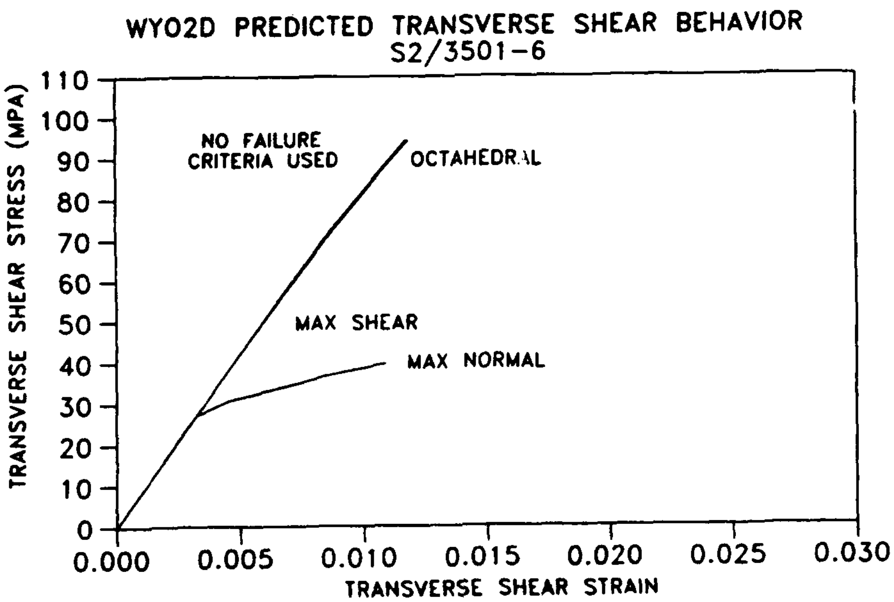


Figure 8. WYO2D predicted transverse shear behavior for S2glass/3501-6 using no failure criteria (material plasticity effects only), and the octahedral, maximum normal stress, and maximum shear stress failure criteria

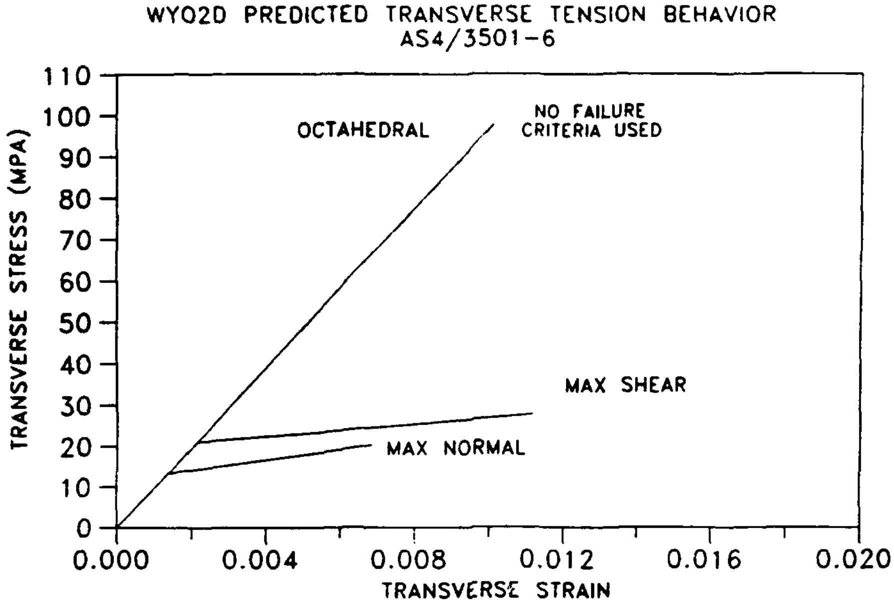


Figure 9. WYO2D predicted transverse tension behavior for AS4/3501-6 using no failure criteria (material plasticity effects only), and the octahedral, maximum normal stress, and maximum shear stress failure criteria

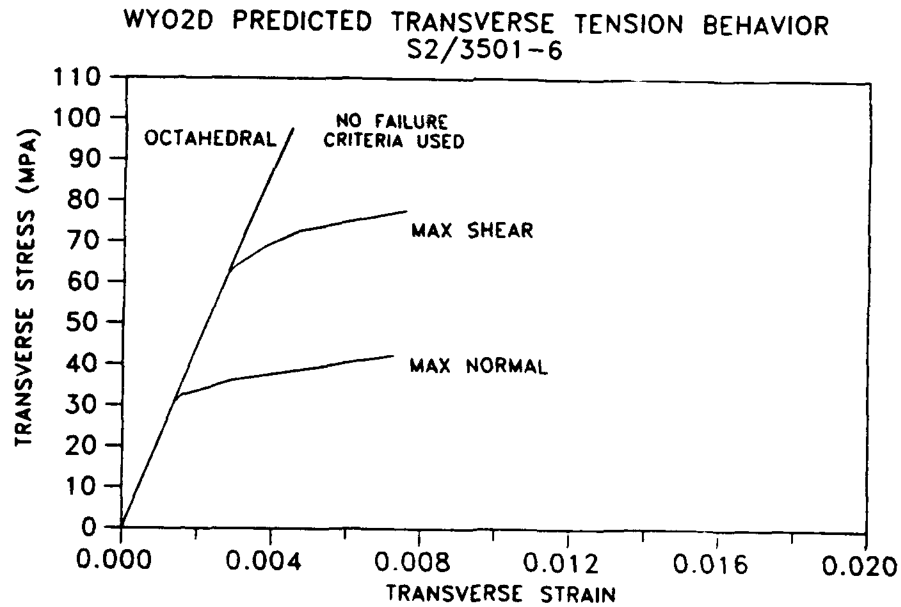


Figure 10. WYO2D predicted transverse tension behavior for S2glass/3501-6 using no failure criteria (material plasticity effects only), and the octahedral, maximum normal stress, and maximum shear stress failure criteria

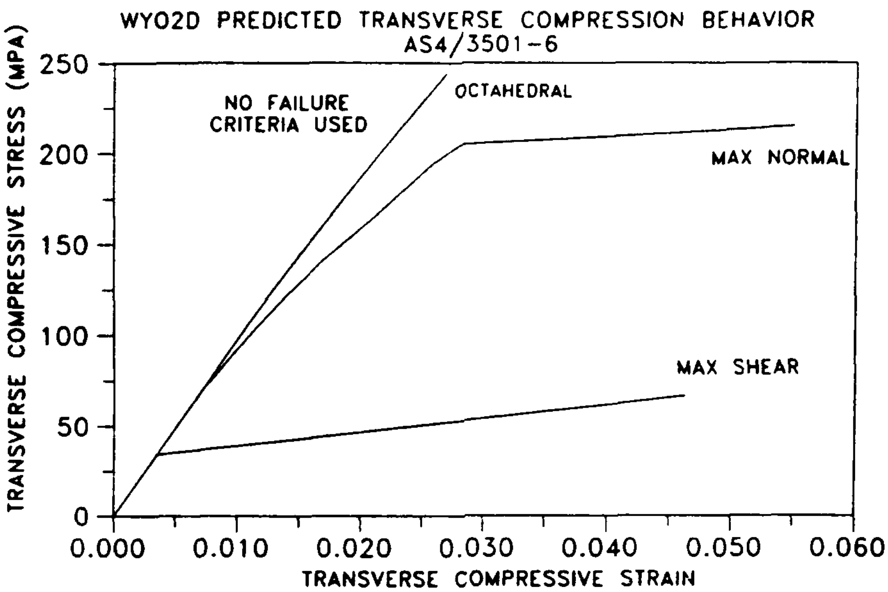


Figure 11. **WYO2D predicted transverse compression behavior for AS4/3501-6 using no failure criteria (material plasticity effects only), and the octahedral, maximum normal stress, and maximum shear stress failure criteria**

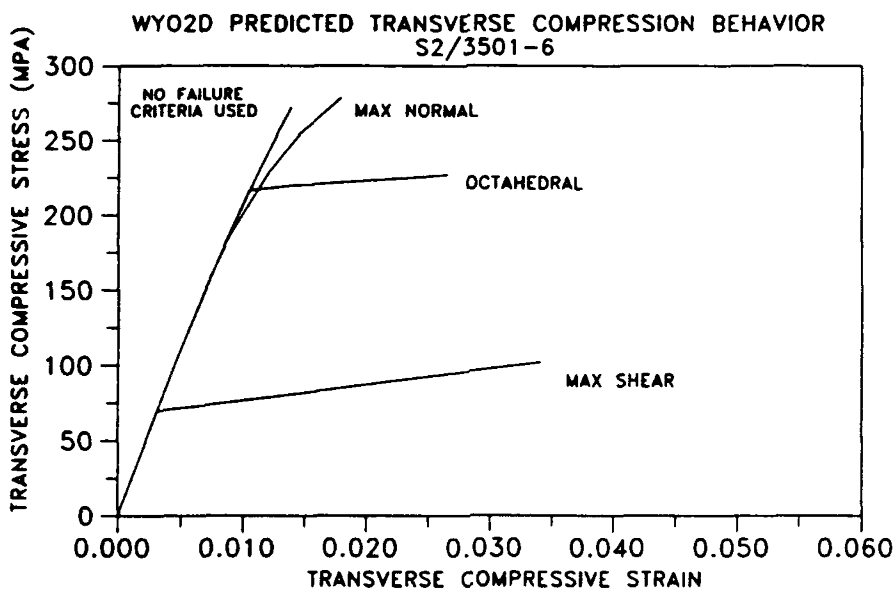


Figure 12. **WYO2D predicted transverse compression behavior for S2glass/3501-6 using no failure criteria (material plasticity effects only), and the octahedral, maximum normal stress, and maximum shear stress failure criteria**

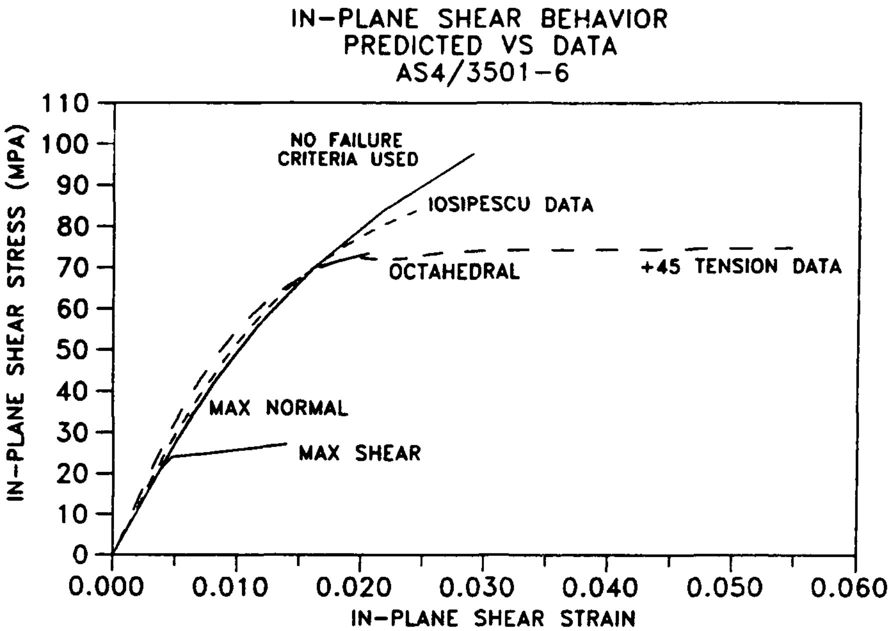
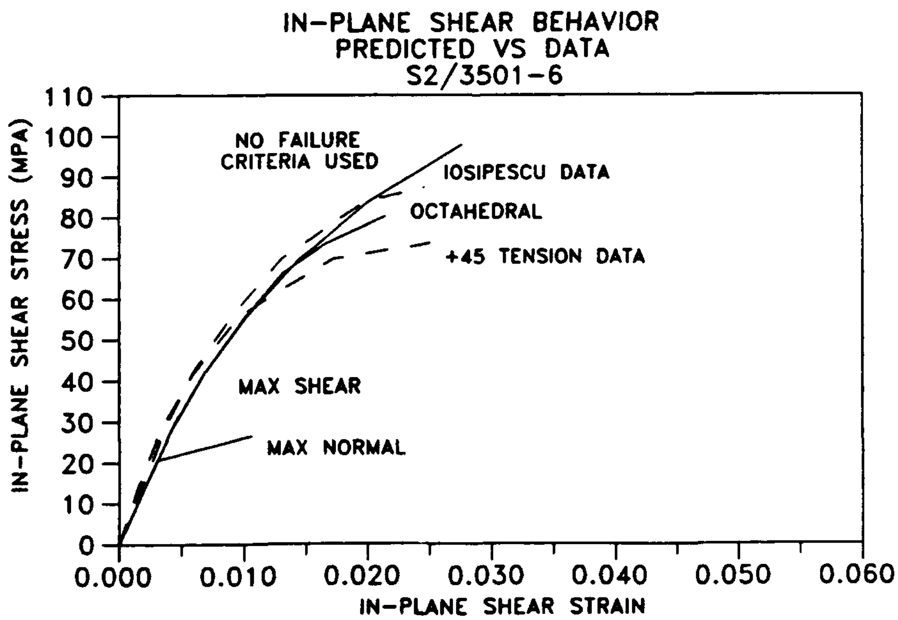


Figure 13. **Comparison of predicted AS4/3501-6 in-plane shear behavior with Iosipescu and ±45° tension shear data**



F: jure 14. **Comparison of predicted S2glass/3501-6 in-plane shear behavior with Iosipescu and ±45° tension shear data**

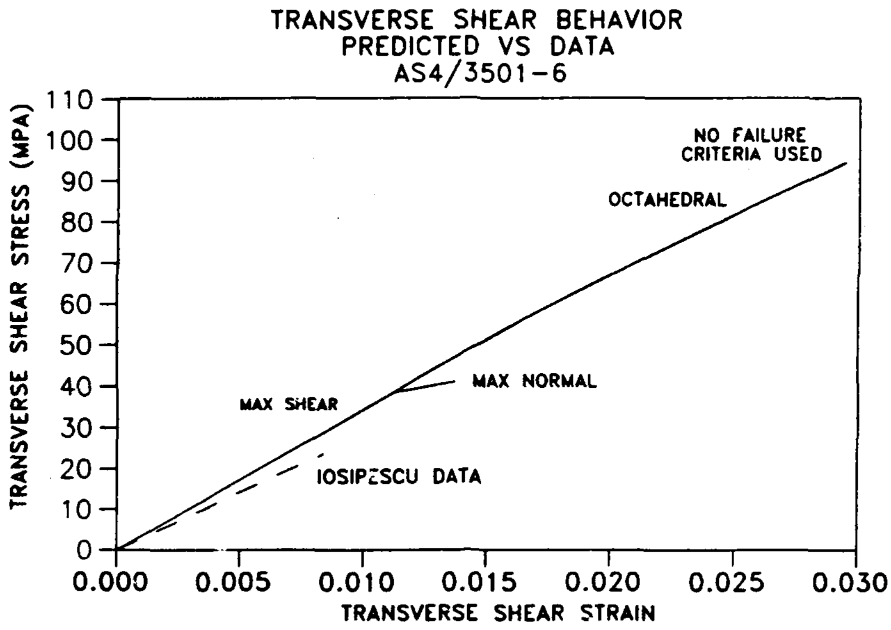


Figure 15. **Comparison of predicted AS4/3501-6 transverse shear behavior with Iosipescu shear data**

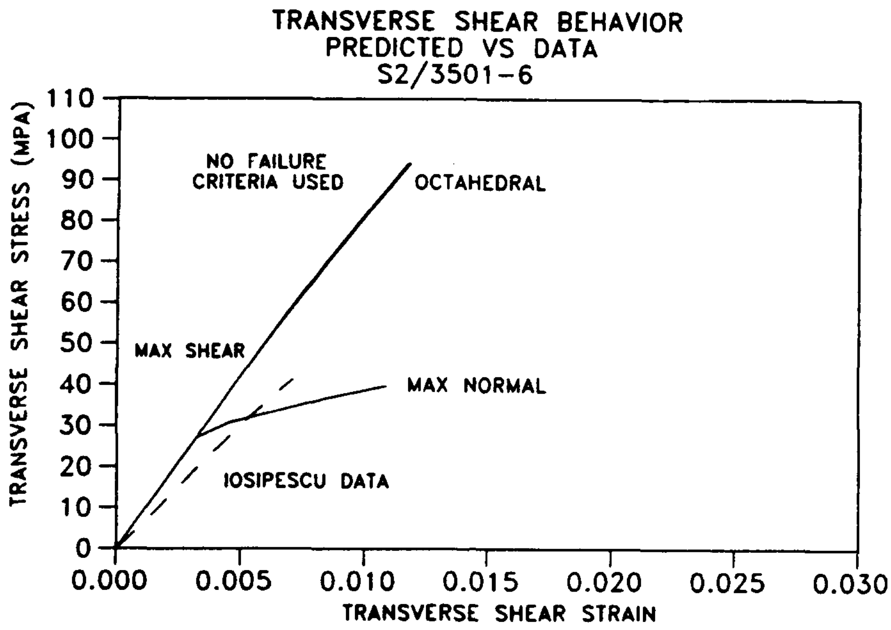


Figure 16. **Comparison of predicted S2glass/3501-6 transverse shear behavior with Iosipescu shear data**

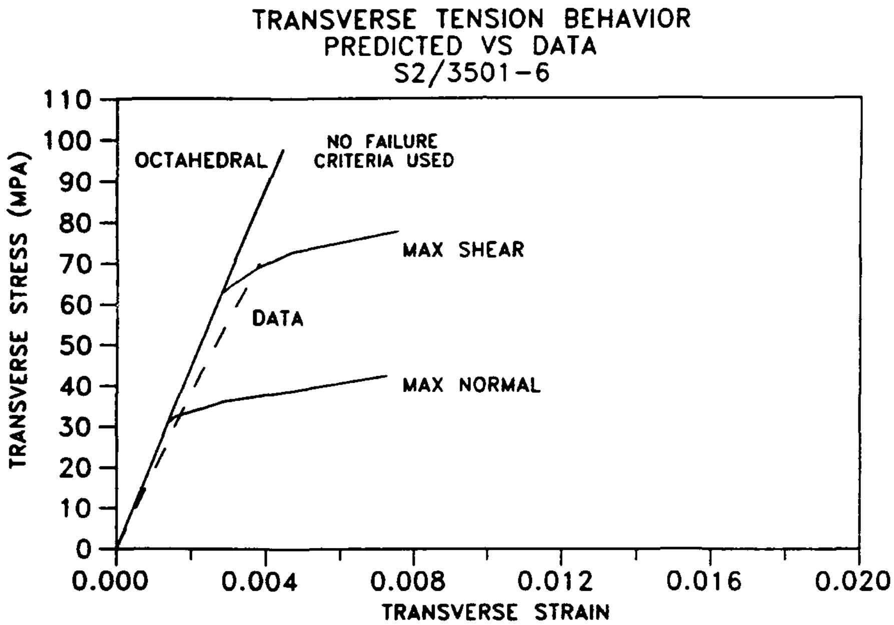


Figure 17. Comparison of predicted S2glass/3501-6 transverse tension behavior with IITRI data

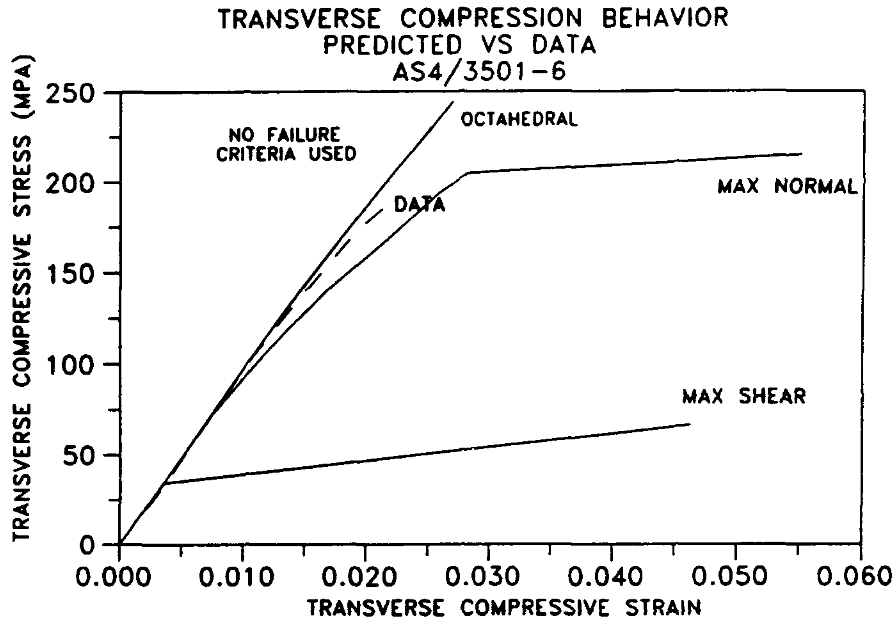


Figure 18. Comparison of predicted AS4/3501-6 transverse compression behavior with data

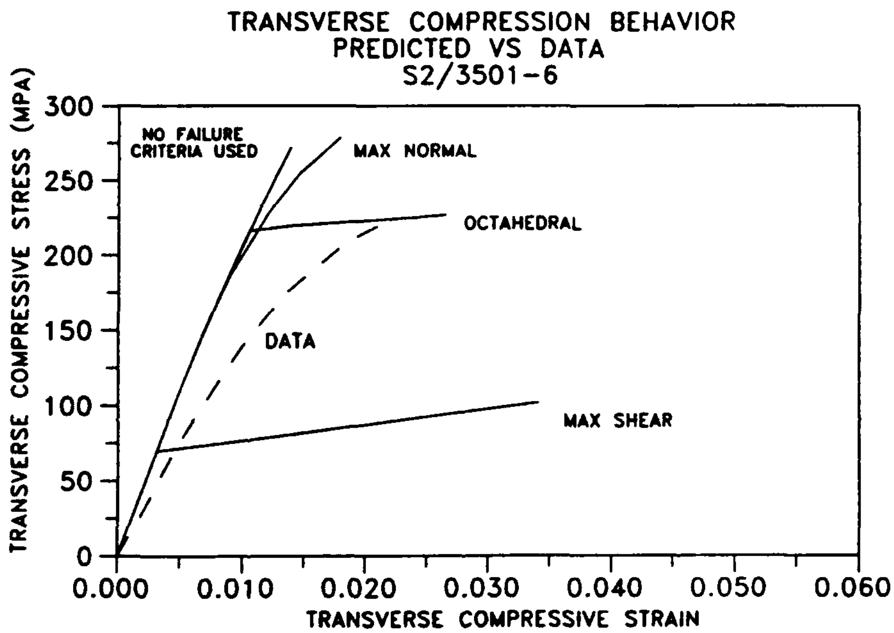


Figure 19. Comparison of predicted S2glass/3501-6 transverse compression behavior with data

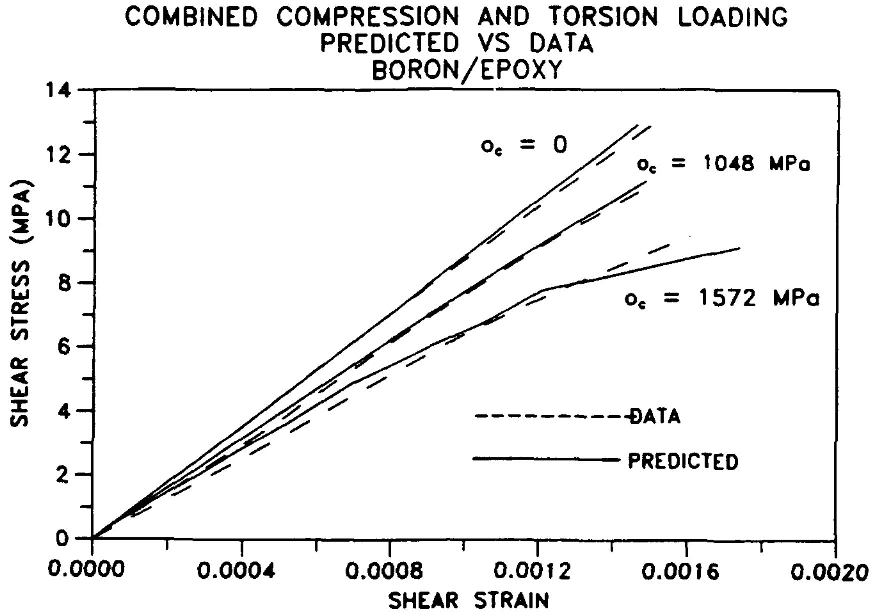


Figure 20. Comparison of predicted boron/epoxy combined compression/shear loading behavior with data from Reference 31

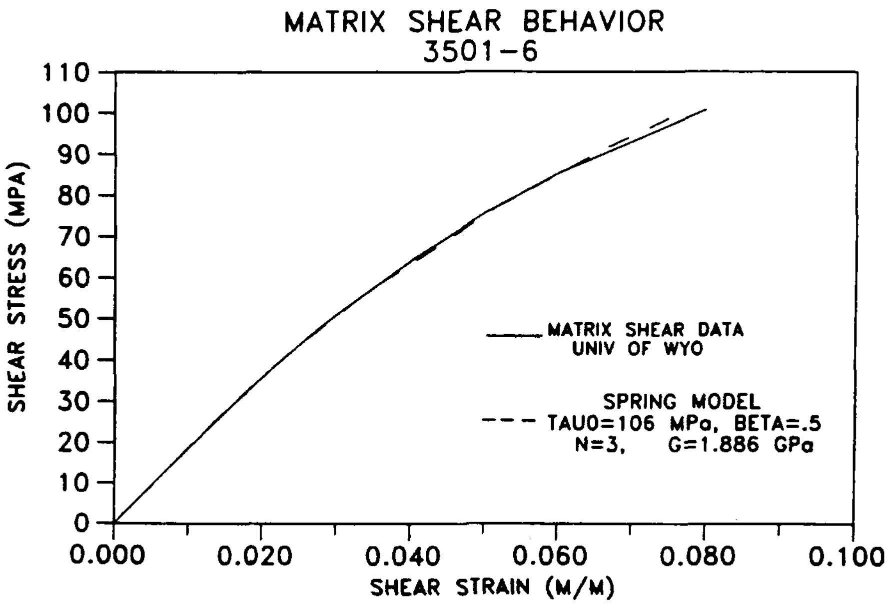


Figure 21. Comparison of matrix shear behavior using empirically based model in WYO2D, and the spring model with matrix parameters chosen so as to match the WYO2D model

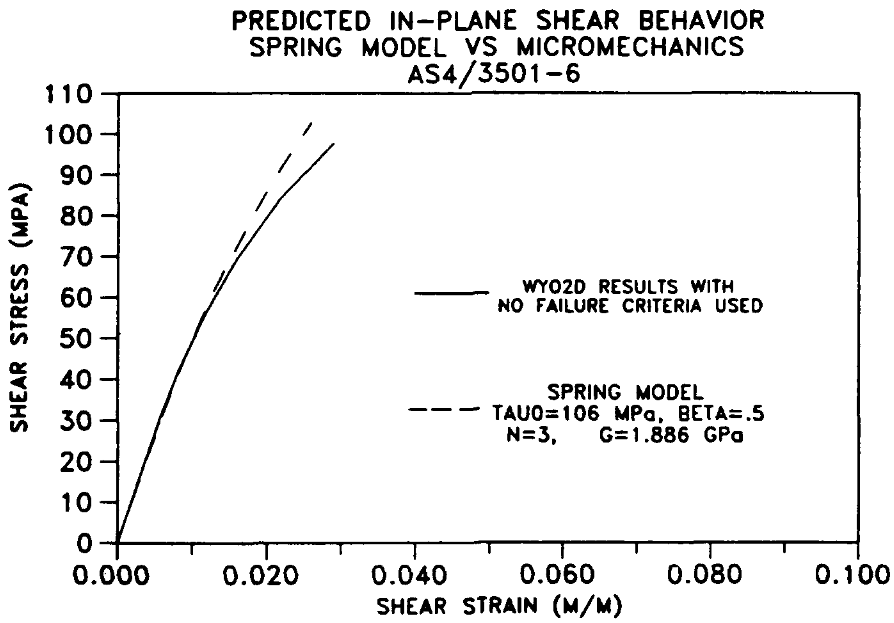


Figure 22. Comparison of predicted AS4/3501-6 in-plane shear behavior using matching matrix models in WYO2d and the spring model

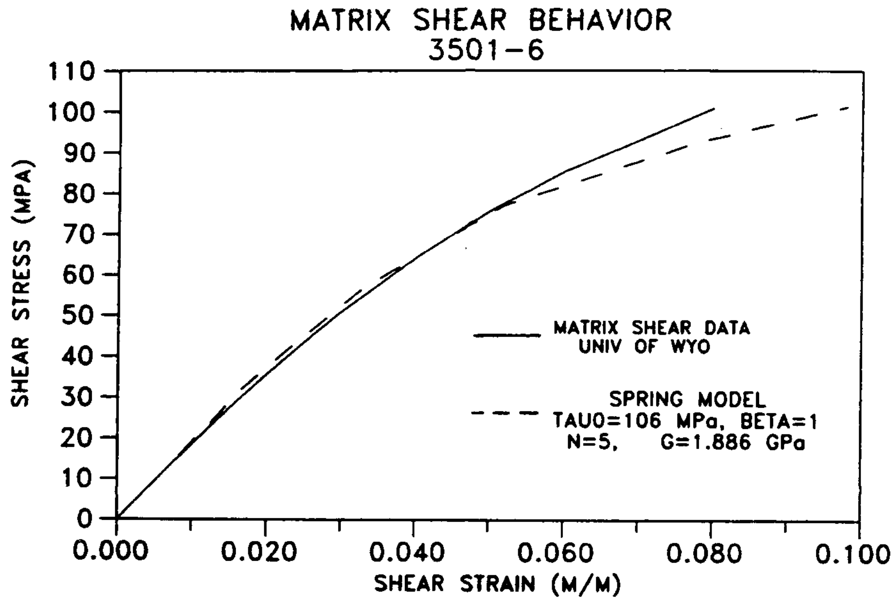


Figure 23. Comparison of matrix shear behavior using empirically based model in WYO2D and revised matrix parameters in the spring model

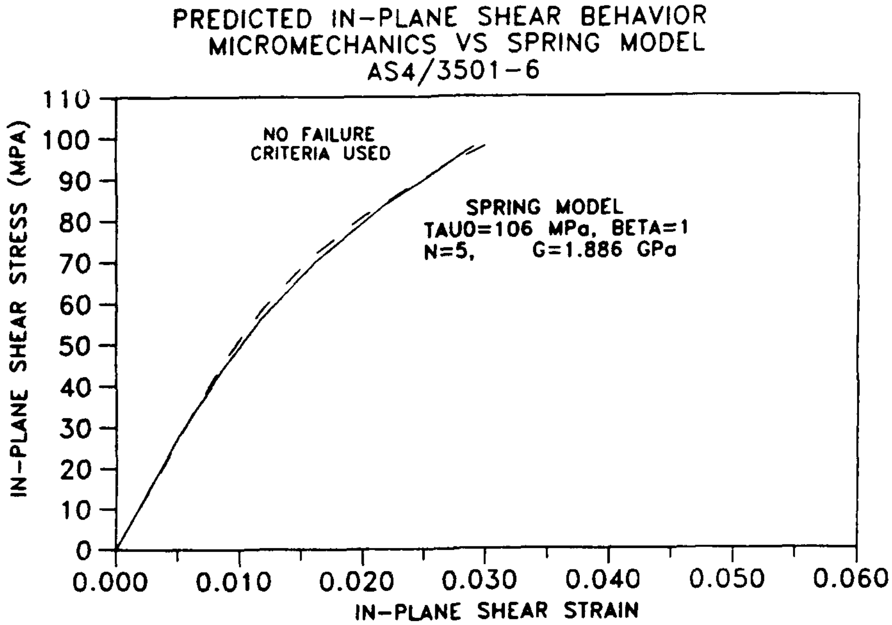


Figure 24. WYO2D and spring model predicted in-plane shear behavior for AS4/3501-6

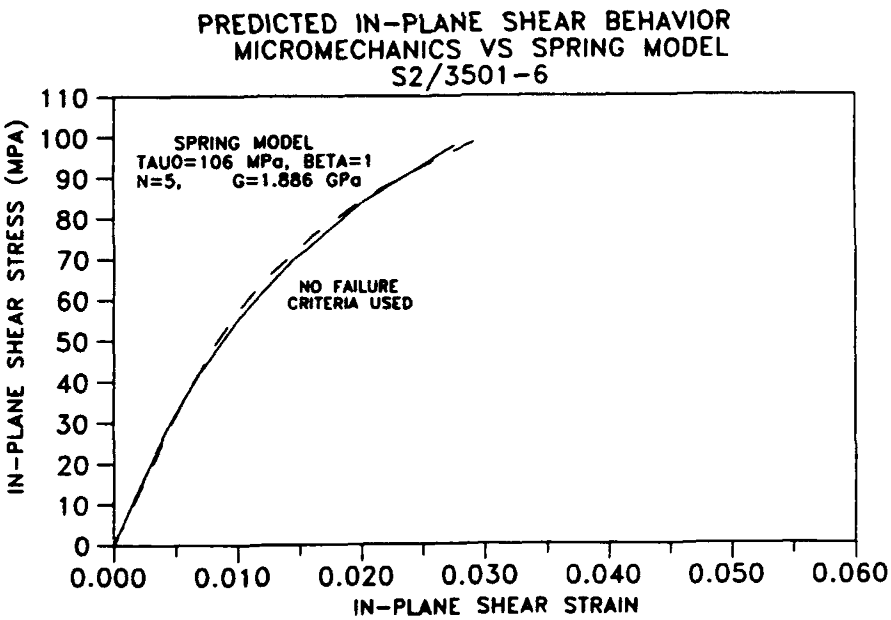


Figure 25. WYO2D and spring model predicted in-plane shear behavior for S2glass/3501-6

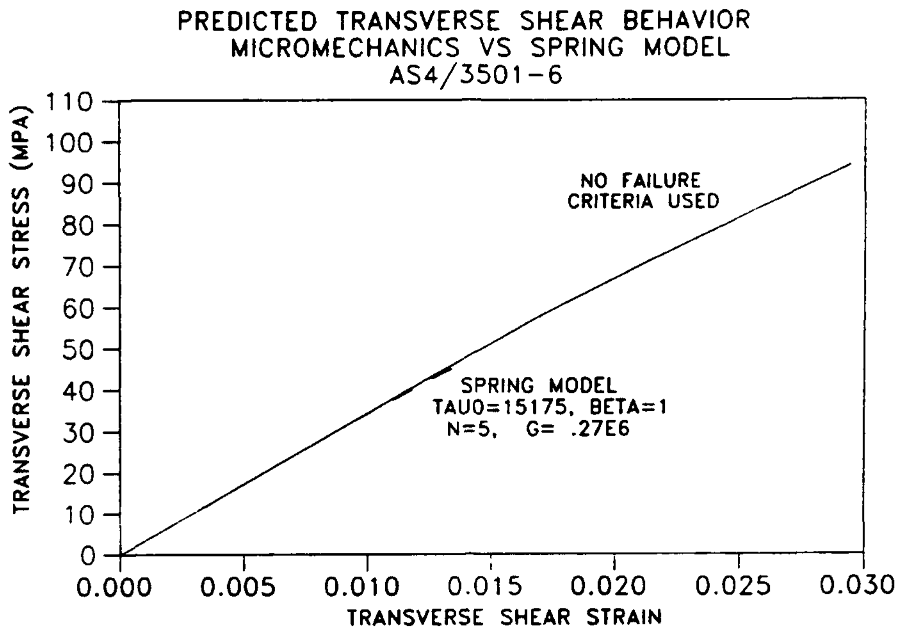


Figure 26. WYO2D and spring model predicted transverse shear behavior for AS4/3501-6

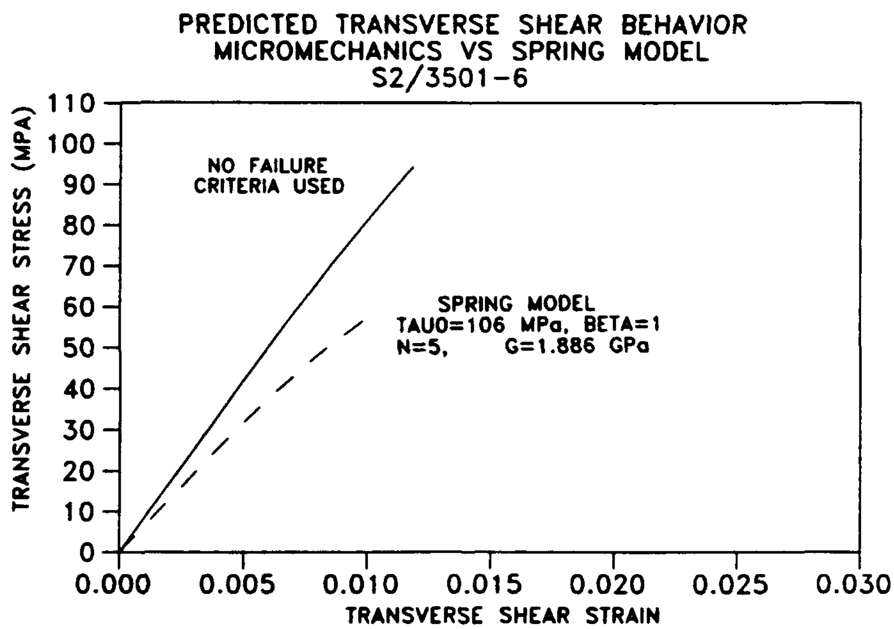


Figure 27. WYO2D and spring model predicted transverse shear behavior for S2glass/3501-6

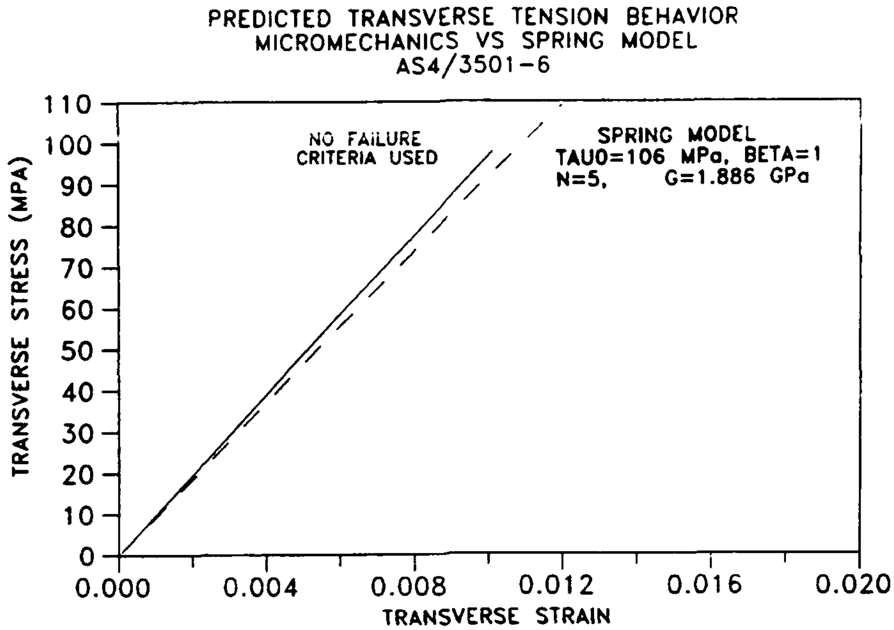


Figure 28. WYO2D and spring model predicted transverse tension behavior for AS4/3501-6

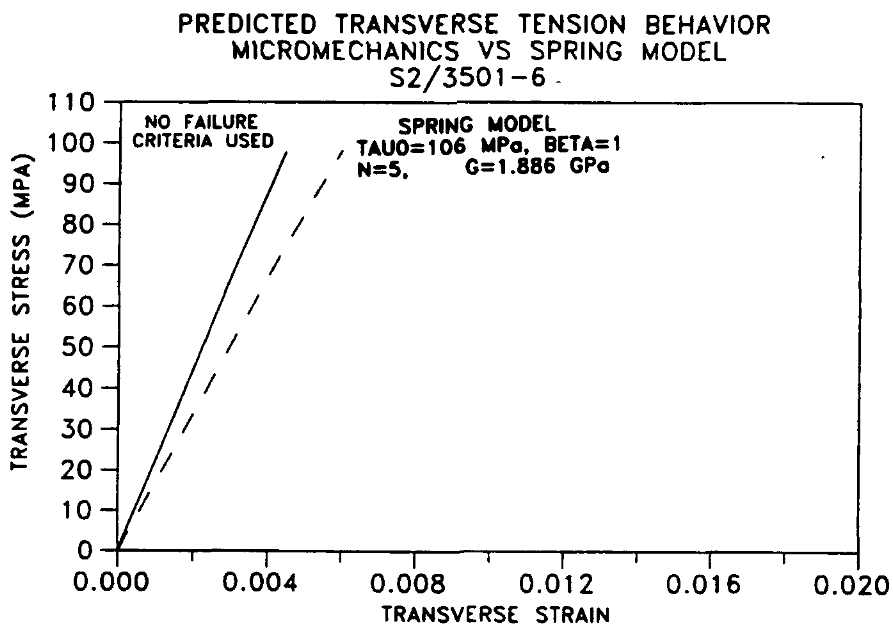


Figure 29. WYO2D and spring model predicted transverse tension behavior for S2glass/3501-6

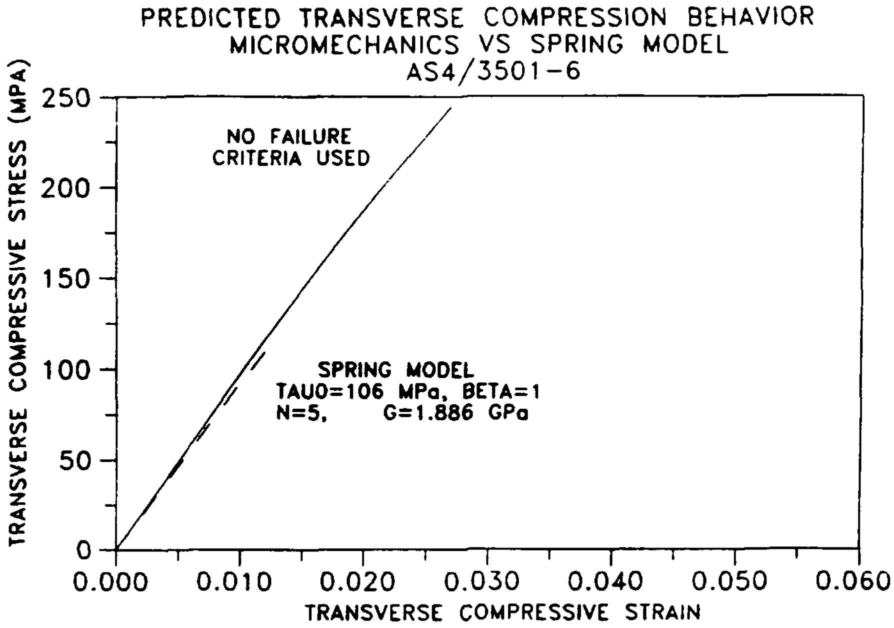


Figure 30. WYO2D and spring model predicted transverse compression behavior for AS4/3501-6

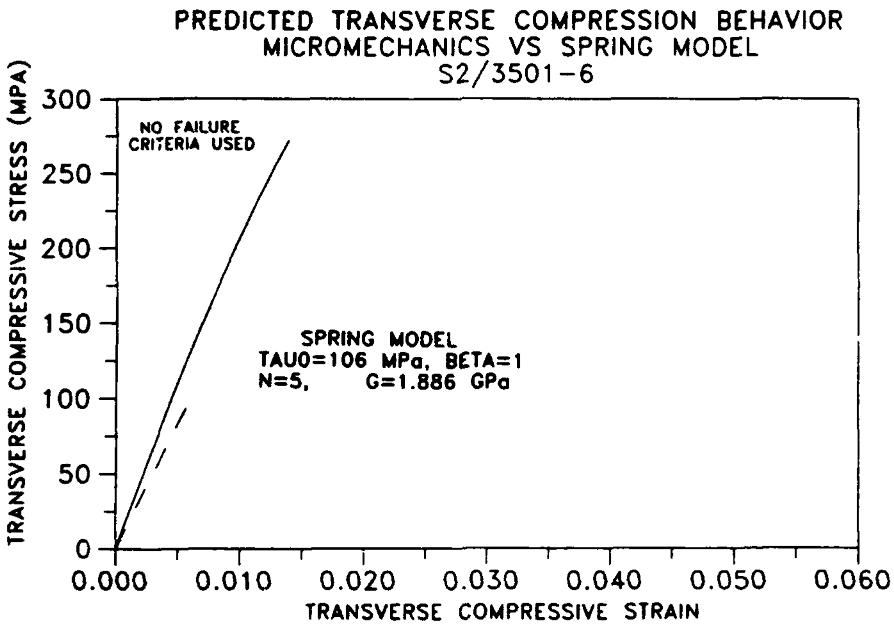


Figure 31. WYO2D and spring model predicted transverse compression behavior for S2glass/3501-6

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13. ABSTRACT *(Maximum 200 words)*

This program outlines a three step process to develop nonlinear composite material models, and incorporate those models in a general, large scale structural finite element analysis code. The steps include detailed finite element analyses at the micromechanical (fiber-matrix) level to study material response and nonlinear mechanisms, development of a simplified material model that is representative of the behavior observed in the micromechanical analyses, and the inclusion of the simplified model in the large scale finite element analysis. Only the first two steps are addressed in this paper. Micromechanical finite element analyses of AS4/3501–6 and S2/3501–6 determined stress-strain responses for in-plane and transverse shear, transverse tension and transverse compression. Variation of the predicted behavior with failure criteria and comparisons to experimental data were also shown. The performance of a simplified material model based on a unit cell analogy was compared to the behavior observed in the micromechanical analyses.

14. SUBJECT TERMS
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